9.2 Surface Areas of Pyramids

**Essential Question** How can you find the surface area of a pyramid?

Even though many well-known pyramids have square bases, the base of a pyramid can be any polygon.

![Pyramids](image)

**Triangular Base**

**Square Base**

**Hexagonal Base**

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**ACTIVITY: Making a Scale Model**

Work with a partner. Each pyramid has a square base.

- Draw a net for a scale model of one of the pyramids. Describe your scale.
- Cut out the net and fold it to form a pyramid.
- Find the lateral surface area of the real-life pyramid.

**a.** Cheops Pyramid in Egypt

Side = 230 m, Slant height ≈ 186 m

**b.** Muttart Conservatory in Edmonton

Side = 26 m, Slant height ≈ 27 m

**c.** Louvre Pyramid in Paris

Side = 35 m, Slant height ≈ 28 m

**d.** Pyramid of Caius Cestius in Rome

Side = 22 m, Slant height ≈ 29 m

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**COMMON CORE Geometry**

In this lesson, you will

- find surface areas of regular pyramids.
- solve real-life problems.

Learning Standard 7.G.6

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Work with a partner. There are many different types of gemstone cuts. Here is one called a brilliant cut.

The size and shape of the pavilion can be approximated by an octagonal pyramid.

a. What does octagonal mean?

b. Draw a net for the pyramid.

c. Find the lateral surface area of the pyramid.

ACTIVITY: Comparing Surface Areas

Work with a partner. Both pyramids have the same side lengths of the base and the same slant heights.

a. **REASONING** Without calculating, which pyramid has the greater surface area? Explain.

b. Verify your answer to part (a) by finding the surface area of each pyramid.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the surface area of a pyramid? Draw a diagram with your explanation.

Practice

Use what you learned about the surface area of a pyramid to complete Exercises 4–6 on page 366.
A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces are triangles. The height of each triangle is the **slant height** of the pyramid.

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**Key Idea**

**Surface Area of a Pyramid**

The surface area $S$ of a pyramid is the sum of the areas of the base and the lateral faces.

$$S = \text{area of base} + \text{areas of lateral faces}$$

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**EXAMPLE 1**

**Finding the Surface Area of a Square Pyramid**

Find the surface area of the regular pyramid.

Draw a net.

<table>
<thead>
<tr>
<th>Area of Base</th>
<th>Area of a Lateral Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 5 = 25$</td>
<td>$\frac{1}{2} \times 5 \times 8 = 20$</td>
</tr>
</tbody>
</table>

Find the sum of the areas of the base and the lateral faces.

$$S = \text{area of base} + \text{areas of lateral faces}$$

$$= 25 + 20 + 20 + 20$$

$$= 105$$

The surface area is 105 square inches.

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**On Your Own**

1. What is the surface area of a square pyramid with a base side length of 9 centimeters and a slant height of 7 centimeters?
EXAMPLE 2
Finding the Surface Area of a Triangular Pyramid

Find the surface area of the regular pyramid.

Draw a net.

\[
\text{Area of Base} \quad \text{Area of a Lateral Face}
\]
\[
\frac{1}{2} \cdot 10 \cdot 8.7 = 43.5 \quad \frac{1}{2} \cdot 10 \cdot 14 = 70
\]

Find the sum of the areas of the base and the lateral faces.

\[
S = \text{area of base} + \text{areas of lateral faces}
= 43.5 + 70 + 70 + 70
= 253.5
\]

The surface area is 253.5 square meters.

EXAMPLE 3
Real-Life Application

A roof is shaped like a square pyramid. One bundle of shingles covers 25 square feet. How many bundles should you buy to cover the roof?

The base of the roof does not need shingles. So, find the sum of the areas of the lateral faces of the pyramid.

\[
\text{Area of a Lateral Face}
\]
\[
\frac{1}{2} \cdot 18 \cdot 15 = 135
\]

There are four identical lateral faces. So, the lateral surface area is

\[
135 + 135 + 135 + 135 = 540
\]

Because one bundle of shingles covers 25 square feet, it will take

\[
\frac{540}{25} = 21.6
\]

bundles to cover the roof.

So, you should buy 22 bundles of shingles.

ON YOUR OWN

2. What is the surface area of the regular pyramid at the right?

3. WHAT IF? In Example 3, one bundle of shingles covers 32 square feet. How many bundles should you buy to cover the roof?
9.2 Exercises

Vocabulary and Concept Check:

1. **VOCABULARY** Can a pyramid have rectangles as lateral faces? Explain.

2. **CRITICAL THINKING** Why is it helpful to know the slant height of a pyramid to find its surface area?

3. **WHICH ONE DOESN'T BELONG?** Which description of the solid does not belong with the other three? Explain your answer.
   - square pyramid
   - regular pyramid
   - rectangular pyramid
   - triangular pyramid

Practice and Problem Solving

Use the net to find the surface area of the regular pyramid.

4. 
   ![Net of a regular pyramid with dimensions 3 in. and 4 in.]

5. 
   ![Net of a regular pyramid with dimensions 9 mm and 10 mm. Area of base is 43.3 mm².]

6. 
   ![Net of a regular pyramid with dimensions 6 m and 6 m. Area of base is 61.9 m².]

In Exercises 7–11, find the surface area of the regular pyramid.

7. 
   ![Regular pyramid with dimensions 9 ft and 6 ft.]

8. 
   ![Regular pyramid with dimensions 6 cm and 4 cm.]

9. 
   ![Regular pyramid with dimensions 10 yd and 9 yd.]

10. 
    ![Regular pyramid with dimensions 10 in., 13 in., and 15 in.]

11. 
    ![Regular pyramid with dimensions 20 mm and 16 mm. Area of base is 440.4 mm².]

12. **LAMPSHADE** The base of the lampshade is a regular hexagon with a side length of 8 inches. Estimate the amount of glass needed to make the lampshade.

13. **GEOMETRY** The surface area of a square pyramid is 85 square meters. The base length is 5 meters. What is the slant height?
Find the surface area of the composite solid.

14.  

15.  

16.  

17. **PROBLEM SOLVING** You are making an umbrella that is shaped like a regular octagonal pyramid.
   a. Estimate the amount of fabric that you need to make the umbrella.
   b. The fabric comes in rolls that are 72 inches wide. You don’t want to cut the fabric “on the bias.” Find out what this means. Then draw a diagram of how you can cut the fabric most efficiently.
   c. How much fabric is wasted?

18. **REASONING** The **height** of a pyramid is the perpendicular distance between the base and the top of the pyramid. Which is greater, the height of a pyramid or the slant height? Explain your reasoning.

19. **TETRAHEDRON** A tetrahedron is a triangular pyramid whose four faces are identical equilateral triangles. The total lateral surface area is 93 square centimeters. Find the surface area of the tetrahedron.

20. **Reasoning** Is the total area of the lateral faces of a pyramid greater than, less than, or equal to the area of the base? Explain.

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**Fair Game Review** What you learned in previous grades & lessons

Find the area and the circumference of the circle. Use 3.14 for \( \pi \).
*(Section 8.1 and Section 8.3)*

21.  
22.  
23.  

24. **MULTIPLE CHOICE** The distance between bases on a youth baseball field is proportional to the distance between bases on a professional baseball field. The ratio of the youth distance to the professional distance is 2:3. Bases on a youth baseball field are 60 feet apart. What is the distance between bases on a professional baseball field? *(Section 5.4)*

   A 40 ft  B 90 ft  C 120 ft  D 180 ft

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